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Entire spatial propagation characteristic of an intense laser beam in plasma : self-focusing and self-trapping due to ponderomotive nonlinearity

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Abstract The self-focusing problem of nonlinear interaction of intense laser beam with plasma has been analysed considering entire spatial characteristics of laser beam Dropping a number of approximations like paraxial approximation and Taylor series expansion of dielectric constant of plasma, self-focusing and self-trapping of the laser beam in plasma with pondermotive nonlinearity has been discussed Comparision of different propagation characteristic parameters like dimensionless beam width self-focusing parameter, self-trapping beam radius and critical power shows a good agreement with moments and varational method The saturation behaviour of equilibrium radius in high intensity region shows better results Value of critical power has been calculated without any kind of power series expansion for the dielectric constant in present approach Results of analysis demonstrate the advantages over popular paraxial ray approximation methods

Keywords Laser-matter interaction, pondermotive nonlinearity, self-focusing and self-trapping

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1. Introduction

An intense laser beam during propagation can modify a nonlinear dispersive medium so as to create a path of enhanced refractive index [1]. The wave refracts into this path and further enhances the nonlinear processes. This can leads to self-similar evolution of laser beam and many important phenomena may occur such as self-focusing, self-trapping *etc*. The recent interest in plasma based particle accelerator concepts and related applications [2,3] has prompted several investigations of self-focusing of laser beam [4–8]. Apart from that, guiding of intense laser beam in plasma channels [9] is beneficial to various applications, including harmonic generation [10], X-ray laser by fiber wave guide scheme [11,12], advance laser fusion scheme [13,14] *etc*. Conventional theories of intense, finite radius pulse propagation in plasma have assumed the paraxial approximation which is incapable of describing many phenomena such as forward Raman scattering [15,16] *etc*. A detailed quantitative understanding of self-focusing is still far away although lot of work had been done since early 1960's [4,5]. A search for effective ways of its analytic description is an on going concern.

Phenomena related to ponderomotive self-focusing of electromagnetic beam in plasma have been considered in the areas of laser driven plasma compression [17], ionospheric modification [18] and heating of magnetically confined plasma column *etc*. The effect has been experimentally verified using an infra-red laser beam in plasma [19]. Theories of self-trapped beams have relied on the paraxial ray approximation [20–23] which is known to give a large error in the value of the threshold power for self-focusing. In view of the possibility that paraxial approximation may also be qualitatively in error in the saturation region, some alternative methods for the analysis of self-focusing and self-trapping of beam in plasma has been suggested and used by various workers [24–28].

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In this paper, we present entire spatial propagation study of intense Gaussian laser beam in plasma for self-focusing and self-trapping. Only ponderomotive nonlinearity in plasma has been considered during the analysis. It is necessary in many paraxial analysis involving self-focusing to make some kind of power series in transverse coordinates and most of it assume to be Taylor series expansion [29 31], that is incorrect. We have applied an alternative method and here self-focusing and self-trapping analysis based on non-Taylor series expansion of dielectric constant of plasma has been presented.

In Section 2, we have solved the wave equation for electric field in nonlinear medium. Here we have derived the beam width parameter differential equation. Equations of self-focusing are given, where expansion of the nonlinear refractive index is not a Taylor series expansion in, radial coordinate (r) as usually employed in paraxial theories [32]. Conditions for diffractionless propagation and self-trapping is presented along with related relations for critical power. Then in Section 3, we have stated the effective dielectric constant of plasma in the presence of intense laser beam when its intensity dependence arises because of ponderomotive force. In Section 4, the numerical results of the analysis based on the present methodology are summarised.

Finally in the Section 5, brief discussion of the results of analysis and their interpretations is given. These results are compared with the available results based on paraxial and non-paraxial approaches such as moments and variational methods. Other related issued are discussed here in this section.

2. Basic equation of laser beam in nonlinear medium

During the development of these equations, it is assumed that time for the establishment of nonlinearity in the medium as well as relaxation time associated with the nonlinearity are much smaller than the pulse width so that steady state expression of the plasma density and refractive index can be used. These assumptions are made throughout so that moving focus phenomena [33] are excluded and the nonlinear response function of the plasma can bear an algebraic relationship with the beam intensity.

2.1. Beam width parameter differential equation for self-focusing :

The wave equation governing the electric vector (E) of the propagating beam in the nonlinear medium can easily be obtained by solving Maxwell's equations and may be written as [34]

$$\nabla^2 E - \frac{\varepsilon}{c^2}\frac{\partial^2 E}{\partial t^2} = 0 . \tag{1}$$

Here, ε is the effective dielectric constant of medium in the presence of the incident beam.

It can in general, be written as

$$\varepsilon = \varepsilon_L + \varepsilon_{NL}(< EE^* >), \tag{2}$$

where ε_L is linear part of the dielectric constant and ε_{NL} is nonlinear part of the dielectric constant of the medium which depends on the intensity of the propagating beam.

A general solution of eq. (1) cannot be obtained. However, it is possible to obtain solution corresponding to common experimental situation of slowly diverging/converging cylindrically symmetric beam which will have a wave-front not very different from a plane wave-front.

Thus, for a azimuthally symmetric beam, eq. (1) can be written as

$$\frac{\partial^2 E}{\partial r^2} + \frac{1}{r}\frac{\partial E}{\partial r} + \frac{\partial^2 E}{\partial z^2} = \frac{1}{c^2}\left(\varepsilon_L + \varepsilon_{NL}\right)\frac{\partial^2 E}{\partial t^2} . \tag{3}$$

The general solution of eq. (3) can be written as

$$E = A(r,z)\exp[i\{\kappa z - \omega t\}] . \tag{4}$$

Substituting eq. (4) into eq. (3), one gets

$$2i\kappa\frac{\partial A}{\partial z} + \left(\frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r}\right) + \frac{\partial^2 A}{\partial z^2}$$

$$= \kappa^2 A - \frac{\varepsilon_L \omega^2}{c^2} A - \frac{\varepsilon_{NL}\omega^2}{c^2} A . \tag{5}$$

But $\kappa = \frac{\omega}{c}\sqrt{\varepsilon_L}$ is propagation constant of the wave and hence above equation can be rewritten as

$$-2i\kappa\frac{\partial A}{\partial z} = \left(\frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r}\right) + \varepsilon_{NL}\frac{\omega^2}{c^2} A \tag{6}$$

This parabolic equation has been extensively employed by various workers for propagation and radiation problem [22].

Putting

$$A(r,z) = A_0(r,z)\exp[-i\kappa S(r,z)] \tag{7}$$

in eq. (6) and separating real and imaginary parts, one gets

$$-2\left(\frac{\partial S}{\partial z}\right) + \left(\frac{\partial S}{\partial r}\right)^2 = \frac{1}{\kappa^2 A_0}\left(\frac{\partial^2 A_0}{\partial r^2} + \frac{1}{r}\frac{\partial A_0}{\partial r}\right) + \frac{\varepsilon_{NL}}{\varepsilon_L} \tag{8}$$

and

$$2\left(\frac{\partial A_0}{\partial z}\right) + 2\left(\frac{\partial A_0}{\partial r}\right)\left(\frac{\partial S}{\partial r}\right) + A_0\frac{\partial^2 S}{\partial r^2} + \frac{A_0}{r}\frac{\partial S}{\partial r} = 0 . \tag{9}$$

Here, S is the plane wave eikonal and $A_0(r,z)$ is the amplitude of the envelope.

For a slowly converging/diverging beam, one can assume the eikonal to be

$$S = \frac{r^2}{2}\beta(z) + \phi(z), \tag{10}$$

also $\beta(z) = \frac{1}{f}\frac{df}{dz}$,

where $[\beta(z)]^{-1}$ represents the radius of curvature of the wavefront and $\phi(z)$ is an addition to the eikonal due to change in the average wave propagation velocity. Here, f, a function of z, is the dimensionless beam width parameter, which is associated with self-focusing behaviour of beam within medium while propagating in the z-direction.

Let us consider the initial intensity distribution of laser beam at plasma-vacuum interface (*i.e.* $z = 0$) along the radial direction, is of Gaussian form such as

$$A_0^2(r, z=0) = E_0^2 \exp\left(-\frac{r^2}{r_0^2}\right) \tag{11}$$

The beam is propagating along the z-direction in the nonlinear medium and its intensity distribution in the medium at any axial distance z, may be given by

$$A_0^2(r,z) = \frac{E_0^2}{f^2(z)} \exp\left(-\frac{r^2}{r_0^2 f^2(z)}\right), \tag{12}$$

where $r_0 f(z)$ represents the spot size of beam at any axial distance z.

Using eqs. (8) and (12), one gets

$$-2\left(\frac{\partial S}{\partial z}\right) + \left(\frac{\partial S}{\partial r}\right)^2 = \frac{r^2}{\kappa^2 r_0^4 f^4}$$

$$\kappa^2 r_r^2 f^2 \quad \frac{\varepsilon_{NL}(<EE^*>)}{\varepsilon_L}. \tag{13}$$

In paraxial approximation, value of eikonal (S) is substituted in this equation and nonlinear part of dielectric constant is approximated as a power series-usually a Taylor series expansion in the transverse cylindrical coordinate r — to estimate the fields near the radial position of interest that is $r = 0$. Finally, r^2 terms on both sides of equation are compared, dropping higher order terms in r and a differential equation for beam width parameter f is obtained. This method, as originally applied by Akhmanov *et al* [22] to self focusing problem and later extended by Sodha *et al* [34] and Max [21] for saturable nonlinearity, does not provide proper results due to accuracy problem [26]. For laser beam in the transverse direction, the plasma density and/or refractive index inhomogenety perceived by the wave profile cannot be approximated by paraxial theories, without proper correction. This is a reason why the conventional paraxial ray theory fails to explain self-focusing properly while the moments and variational methods intrinsically take care of these approximation.

For the study of entire spatial beam self-focusing including paraxial as well as pheripheral portion of the axially peaked laser beam (Gaussian), value of eikonal (S) from eq. (10) is substituted in eq. (13) and without expanding the nonlinear dielectric constant and dropping higher order term, one obtains the equation for beam width parameter as

$$\frac{d^2 f}{dz^2} = \frac{2}{\kappa^2 r_0^2 r^2 f} - \frac{1}{\kappa^2 r_0^4 f^3} - \frac{f}{r^2}\frac{\varepsilon_{NL}}{\varepsilon_L}(<EE^*>). \tag{14}$$

The above equation is the second order differential equation for the beam width parameter (f) with r and z as variable. It defines the propagating beam dynamics and can be used for study of self-focusing of the beam with arbitrary cross section in all types of nonlinear medium. In the eq (14), first two terms of the R.H.S. are responsible for diffraction divergence effect while the last term corresponds to convergence effect due to nonlinear refraction of the beam.

2.2 *Diffractionless propagation and normalised self-trapping radius :*

During the propagation of intense laser beam in nonlinear medium, diffraction effect continuously competes with focusing effect, governing the propagation characteristics of the beam and its dynamics. We shall discuss here condition, under which an electromagnetic beam can produce its own dielectric wave-guide and propagate without spreading. Such self-trapping or diffractionless propagation in dielectric wave-guide mode appears to be possible for intense laser beams where dielectric constant increases with field intensity of beam. This phenomena of self-trapping can produce marked optical and physical effect.

Following eq. (14), one can conclude that when the diffraction divergence of laser beam is exactly balanced by the focusing effect in the medium due to different type of nonlinear effect, the beam propagates in a self-trapped wave-guide mode without convergence or divergence. Thus for an initial plane wave-front of the beam, at $z = 0, f = 1$, as well as $\frac{df}{dz} = 0$ and hence $\frac{d^2 f}{dz^2} = 0$. This leads to a condition, where the terms in the right hand side of eq. (14) cancel each other. In this situation, the beam width of laser does not change during the propagation in the nonlinear medium. In other words, the beam propagates without convergence or divergence or in the self-trapped mode.

On applying these conditions in eq. (14), one obtains

$$\frac{2}{\kappa^2 r_0^4} \quad \frac{1}{\kappa^2 r_0^4} \quad \frac{\varepsilon_{NL}(<EE^*>)}{\varepsilon_L r_0^2} = 0$$

$$\frac{}{\kappa^2 r_0^2} \quad \frac{\varepsilon_{NL}(<EE^*>)}{\varepsilon_L}.$$

But the propagation constant $\kappa = \frac{\omega}{c}\sqrt{\varepsilon_L}$, so this equation can be rewritten as

$$\frac{\omega r_0}{c} = \{\varepsilon_{NL}\}^{-\frac{1}{2}}(z = 0, f = 1). \tag{15}$$

2.3. *Critical power :*

The critical power of beam P_{cr} is one of the important parameter in self-focusing problem. It is defined as the minimum power of the incident beam for which the beam propagates in nonlinear medium without converging or diverging *i.e.* in uniform wave-guide mode. In general, it is the minimum power of incident beam required to create a self-focused channel.

The critical power of beam in the nonlinear medium such as plasma, is given as [34]

$$P_{cr} = \frac{\varepsilon_L}{4} \frac{c}{\varepsilon_L^{\frac{1}{2}}} \int_0^{\infty} \frac{E_0}{f^2} \exp\left[-\frac{r^2}{r_0^2 f^2} \right] r \, dr$$

$$\frac{c}{8} \varepsilon_L^{\frac{1}{2}} r_0^2 E_{ocr}^2, \tag{16}$$

where $E^2{}_{ocr}$ is square of the critical value of electric field amplitude and can be calculated using the self-trapping condition (15).

The eq. (16) show that the critical power depends on the size of the incident beam and critical value of electric field amplitude. Incident beam, above the critical power level may be trapped at any arbitrary diameter and not spread unless some instability and related phenomena forces it to do so.

3. Effective dielectric constant of the plasma in the presence of intense laser beam : ponderomotive nonlinearity

Let us consider that the ponderomotive nonlinearity in collisionless plasma is mainly responsible for nonlinear dielectric constant. It arises due to interaction of electron with the magnetic field of the propagating beam. This is considered here because the ponderomotive nonlinearity with relaxation time $\tau_p \simeq 10^{-9}$ to 10^{-11} sec, sets much quicker. The laser pulse of width τ such that $\tau > \tau_p$ are sensitive only to the ponderomotive mechanism. In most of experiments conducted with high power laser, this condition $\tau > \tau_p$ is satisfied.

Following Anderson and Bonnedal [28], the nonlinear part of the dielectric constant of plasma for ponderomotive nonlinearity in the presence of intense laser beam can be written as

$$\varepsilon_{NL}(EE^*) = \frac{\omega_p^2}{\omega^2}\left[1 - \exp\left\{-\frac{3m}{4M}\alpha EE^*\right\}\right]. \tag{17}$$

Here, $\omega_p = \left[\frac{4\pi N_0 e^2}{m}\right]^{\frac{1}{2}}$ is the plasma frequency is the absence of the laser beam. N_0 and e are density and charge of electron. The characteristic parameter α is given as

$$\alpha = \frac{e^2 M}{6 k_B T_0 m^2 \omega^2}.$$

At the plasma-vacuum interface *i.e.* $z = 0, f = 1$, the nonlinear part of the dielectric constant of plasma, without use of Taylor series expansion is

$$\varepsilon_{NL} = \frac{\omega}{\omega}\left[1 - \exp\left\{-\frac{3m}{4M}\alpha E_0^2 \exp\left(-\frac{r^2}{r_0^2}\right)\right\}\right]. \tag{18}$$

Combining eqs. (2) and (18), the general expression for the effective dielectric constant of plasma at $z = 0$ due to ponderomotive nonlinearity can be written as

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2} + \frac{\omega_p^2}{\omega^2}\left[1 - \exp\left\{-\frac{3m}{4M}\alpha E_0^2 \exp\left(-\frac{r^2}{r_0^2}\right)\right\}\right] \tag{19}$$

This expression of the effective dielectric constant will be used with self-focusing equation during analysis presented in Section 4.

Figure 1 where nonlinear part of dielectric constant due to ponderomotive force in plasma is plotted as a function of radial distance from the axis, shows a gradual decrease in the

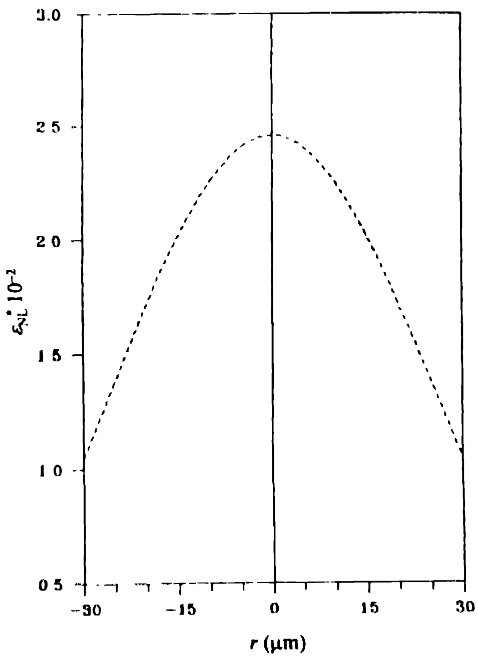

Figure 1. Radial profile of nonlinear part of dielectric constant (ε_{NL}) of plasma for ponderomotive nonlinearity. Here r_0 = 30 μm, βE_0^2 = 0.55, ω_p = 2.5 × 10^{13} rad/sec and ω = 1.0 × 10^{14} rad/sec.

nonlinear part of dielectric constant as one moves away from the axis in radially outward direction. This decrease is associated to plasma density inhomogeneity perceived by the transversely inhomogeneous wave field.

Following eq. (17), one can obtain

$$\varepsilon_{NL} = \frac{\omega_p^2}{\omega^2} \frac{\exp\left\{\frac{3m}{4M}\alpha EE^*\right\} -}{\exp\left\{\frac{3m}{4M}\alpha EE^*\right\}} \tag{20}$$

It indicates that the nonlinear part of dielectric constant due to ponderomotive nonlinearity, shows a saturation behaviour as the beam intensity increases to a very high values. Laser beam intensity near the focal point will increase to high enough level which will be capable of displaying saturation effects as shown in Figure 2 and value of nonlinear part of dielectric constant corresponding to beam intensity parameter is tabulated in Table 1.

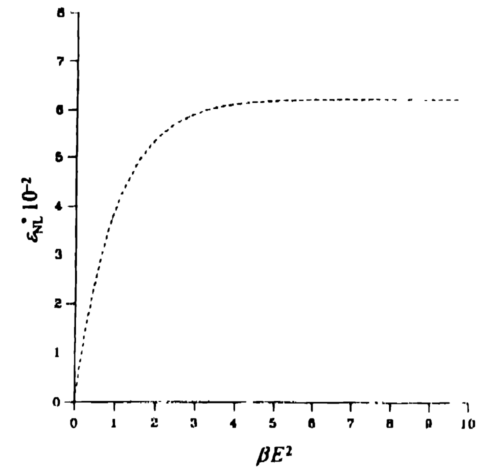

Figure 2. Nonlinear part of dielectric constant (ε_{NL}) variation with the beam intensity parameter (βE_0^2) for ponderomotive nonlinearity in plasma. $\beta = \frac{3m\alpha}{4M}$

Table 1. Nonlinear part of the dielectric constant (ε_{NL}) of plasma for ponderomotive nonlinearity for different value of beam intensity (βE_0^2). $\omega_p = 2.5 \times 10^{13}$ rad/sec $\omega = 1 \times 10^{14}$ rad/sec and $r_0 = 30$ μm

βE_0^2 (in arb Unit)	$\varepsilon_{NL}^{\bullet}\ 10^{-2}$
1	3 95
2	5 50
3	5 94
4	6 14
5	6 21
6	6 23
7	6 25
8	6 25
9	6.25
10	6.25

4. Numerical results

The foregoing analysis of laser beam propagation in plasma has been used to obtain many important results. For ponderomotive nonlinearity, the value of effective dielectric constant from eq. (17) is substituted in the eq. (14) and resulting self-focusing equation is given as

$$\frac{d^2f}{dz^2} = \frac{1}{\kappa^2 r_0^2 r^2 f} - \frac{1}{\kappa^2 r_0^4 f^3} - \frac{f}{r^2}\frac{\omega_p^2}{\omega^2 \varepsilon_L}\left[1-\exp\left\{-\frac{3m}{4M}\frac{\alpha E_0^2}{f^2}\exp\left(-\frac{r^2}{r_0^2 f^2}\right)\right\}\right]. \quad (21)$$

It is a second order differential equation of dimensionless beam width parameter (self- focusing parameter) which is a function of both r and z.

The main feature of this equation is that it provides the beam width inside the plasma at any axial distance for any arbitrary cross section of incident beam. Hence, it can be used to obtain the entire spatial propagation characteristic of laser beam in plasma.

A numerical solution of this equation has been obtained for a typical sample plasma with the following parameters :

ω (frequency of incident beam) = 1×10^{14} rad/sec,

ω_p (plasma frequency) = 2.5×10^{13} rad/sec,

T_0 (temperature of plasma) = 10^5 k,

r_0 (initial size of laser beam) = 30 μm,

N_0 (electron number density) = 9.5×10^{17} cm^{-3} and,

βE_0^2 (initial intensity parameter) = $\frac{3m}{4M}\alpha E_0^2 = 0.55$.

Runge-Kutta method is used to solve the differential eq. (21) with above listed value for different parameters. The results are shown in Figure 3 (curve A) and tabulated in Table 2. The variations of f with z demonstrate an oscillatory behaviour

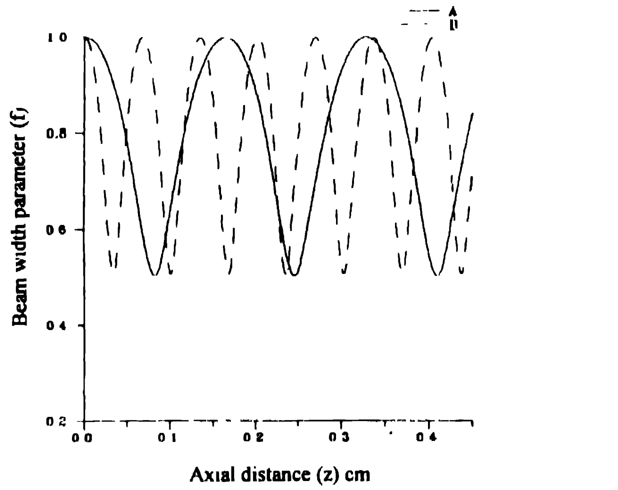

Figure 3. Oscillatory behaviour of self focusing beam width parameter (f) with axial distance (z) for ponderomotive nonlinearity. Curve (A) for present entire spatial beam analysis (non paraxial) and curve (B) for paraxial method $\omega_p = 2.5 \times 10^{13}$ rad/sec, $\omega = 1 \times 10^{14}$ rad/sec, $\beta E_0^2 = 0.55$ and $r_0 = 30$ μm

Table 2. Minimum value of beam width parameter (f_{min}) and corresponding axial position (Z_{min}) for paraxial method and present non-paraxial analysis $\omega_p = 2.5 \times 10^{13}$ rad/sec, $\omega = 1 \times 10^{14}$ rad/sec, $\beta E_0^2 = 0.55$ and $r_0 = 30$ μm

Method	First minima		Second minima		Third minima	
	$f_{min}(1)$	$Z_{min}(1)$ cm	$f_{min}(2)$	$Z_{min}(2)$ cm	$f_{min}(3)$	$Z_{min}(3)$ cm
Present non-paraxial analysis	0.5036	0.082	0.5037	0 246	0.5037	0.410
Paraxial method*	0.5074	0 034	0.5073	0.102	0.5078	0.169

*Using Ref. [21]

which indicates that during the popagation, the laser beam aperture in plasma first decreases and attaining the minimum value, it increases. This process repeats again and again providing oscillatory behaviour. Mathematically, these results shows that at $z = 0$, value of $f = 1$ and $df/dz = 0$ *i e.* the beam width has no initial divergence. With the increase in the value of z in the vacinity of $z = 0$, df/dz becomes negative and f starts decreasing and its value becomes less than unity. The first two terms in the R.H.S. of eq. (21) decrease more rapidly than the third term. At $z = z_{min}$, f attains a minimum value *i.e.* $f_{min} = 0.5036$ for $z_{min} = 0.082$ (see Table 2). At this point, axial intensity of the focused beam is considerably enhanced and thus d^2f/dz^2 becomes positive. Beyond $z = z_{min}$, df/dz takes positive value and thus f starts increasing. In Figure 3 the results obtained from paraxial theory [21] are also represented as curve B, for comparison. It is clear from Figure 3 that present theory predicts large value of self-focusing distance as compared to conventional paraxial theory which is also reflected in the higher value of z_{min} in Table 2. Self-focusing parameters obtained by paraxial and our method are almost same, slightly less for present non-paraxial analysis.

In the analysis, incident axial symmetrical laser beam profile assumed to be Gaussian. For the different spatial position (*i.e.* distance from the axis of propagating beam), self-focusing parameter (f) as a function of axial distance (z) in plasma has been numerically obtained and represented in Figure 4. Results indicate that the trajectories of rays emerging from differential spatial position in the Gaussian beam, are different from one another. Result obtained from paraxial method [21] employing same sample parameters, are also shown in Figure 4. Careful study of Figure 4 shows that the

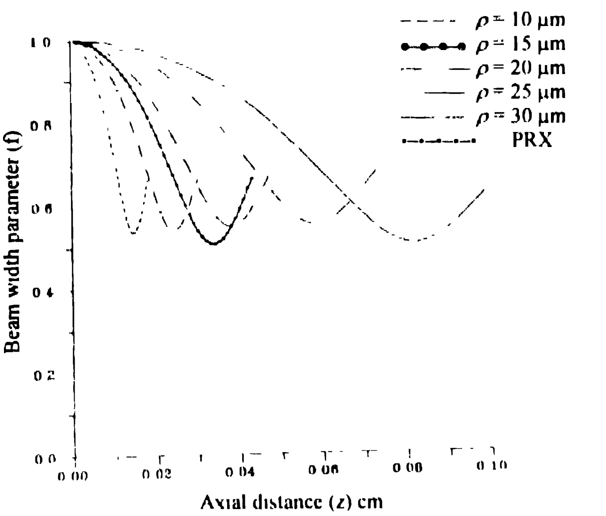

Figure 4. Axial dependence of beam aperature (abiratary units) for different spatial distance (ρ). Curve 1 to 4 are for ρ = 10 µm, 15 µm, 20 µm, 25 µm and 30 µm respectively r_0 = 30 µm, ω_p = 2.5 × 10^{11} rad/sec, ω = 1 × 10^{14} rad/sec

focal length of the paraxial part of the beam is smaller than the peripheral part, indicating aberration effect. In comparison to it, paraxial methods which are valid only for self-focusing of near axis part of the beam, gives only one focal point [22].

Beam trapping without diffractional divergence is an important topic of self-focusing problem. According to present analysis, self-trapping condition for ponderomotive nonlinearity in plasma can be obtained by sbstituting eq. (18) in eq. (15). For spatial position $r = \rho$, the normalised self-trapped radius ($\omega_p\rho/c$) becomes :

$$\frac{\omega_p\rho}{c} = \left(2r_0^2 - \rho^2\right)^{\frac{1}{2}} \frac{\rho}{r_0^2} \times\left[1 - \exp\left\{-\beta E_0^2 \exp\left(-\frac{\rho^2}{r_0^2}\right)\right\}\right]^{-\frac{1}{2}}. \tag{22}$$

Using this relation, self-trapped radius has been calculated for different values of initial beam intensity parameter (βE_0^2) and different spatial beam positions. These results are represented in Figure 5 for normal self-trapping. Study of the plots in Figure 5 indicates that the paraxial portion of the spatial cross-section of the incident Gaussian beam corresponds to small trapping radius as compared to peripheral portion *i.e.* for higher value of ρ. In the lower power region, the vertical portion of the curves in Figure 5 shows that the self-trapping is possible for a certain value of beam power, known as critical power. It is different for paraxial and

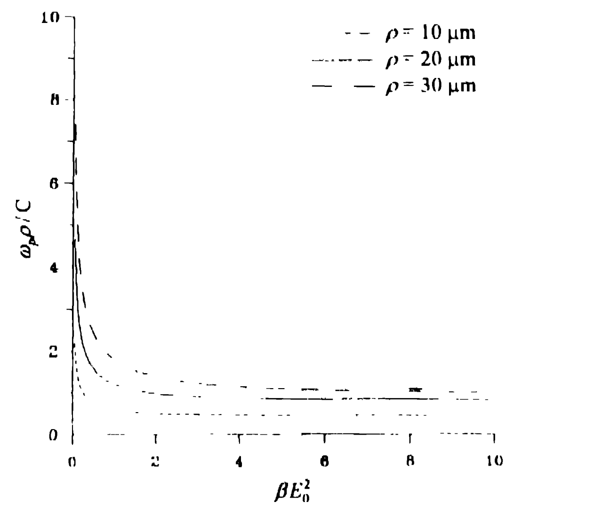

Figure 5. Normalised self-trapped radius of beam ($\omega_p\rho/c$) dependence on beam intensity parameter (βE_0^2) for different value of spatial distance (ρ)

peripheral portions of beam. Results also indicate that saturation behaviour starts for higher value of beam intensity for larger value of ρ. Calculated value of critical power from the present analysis [P_{cr}(NPR)] for different spatial position coordinates is given in Table 3. Results are compared with the paraxial method [21] and ratio of critical power from our method to the paraxial method is shown in the last column of this table. The intensity profile of the laser beam in the transverse direction will be modified due to induced inhomogeneity in plasma density or refractive index.

Table 3. Critical power (P_{cr}) corresponding to different spatial position of incident beam cross section considering ponderomotive nonlinearity in plasma and its ratio to paraxial method value $\omega_p = 2.5 \times 10^{13}$ rad/sec, $\omega = 1.0 \times 10^{14}$ rad/sec and $r_0 = 30$ μm

Spatial distance μm	P_{cr} Kw	Critical power ratio P_{cr}/P_{cr}^{*} (paraxial)
10	279 27	1 00
15	321 13	1 15
20	390 53	1 40
25	502.38	1 80
30	683 77	2.46

*Using Ref [21]

5. Discussion

The key idea of our approach to self-focusing problem is not to involve Taylor series or any kind of power series expansion for dielectric constant as usually done in many paraxial analysis [35]. Hence, the results of the present analysis are not confined to only near axis parts of the beam but deals with entire spatial propagation characteristic of assumed intense Gaussian laser beam.

Observed oscillatory behaviour of beam aperture during the propagation of laser beam in nonlinear plasma medium in axial direction may be due to the fact that in the vacinity of plasma vacuum interface *i.e.* $z = 0$, with the increasing value of z, diffraction divergence decreases more rapidly than nonlinearity based convergence or focusing effect and consequently, beam aperture decreases. Due to continuous decrease in beam aperture, at particular value of $z = z_{min}$, intensity of the beam is considerably enhanced and diffraction divergence starts dominating over focusing convergence effect. Thus after attaining the minimum value, beam aperture starts increasing beyond z_{min} *i.e.* $z > z_{min}$. After propagation of certain length, beam aperture decreases to a value where focusing effect starts dominating the defocusing diffraction effect and beam aperture starts decreasing again. Because of these two diffraction and nonlinearity related self focusing effects and their dominance over one another during the propagation of laser beam in axial direction, medium acts as an oscillatory wave-guide.

Results shown in Figure 4, represent strong aberrational effect in the self focusing behaviour when entire spatial beam is considered in the analysis. In the paraxial method, after the Taylor series expansion only aberrationless parts in the nonlinear refraction force, which are quadratic in the coordinate r, are retained. Hence, such methods predicts only one focus for the self-focusing. The aberrationless approximation is valid only for self focusing behaviour of near axis parts of the beam far from the realistic situation. Present analysis indicates that under axial symmetry, a structure of circular zones arises similar to that caused by spherical aberration of ordinary lenses. This spherical aberration, which appears to be inherent in self-focusing, may lead to deviations from axial symmetry, similar to the astigmatism of ordinary lenses.

The beam critical power P_{cr} is one of the important parameter in self-focusing and self-trapping problem. Results of present analysis presented in Table 3, show very interesting behaviour for critical power. For the near axis region where spatial distances $\rho = 10$ μm, the value of critical power for the plasma medium of specific parameters (used in the present numerical analysis) is 279.27 Kw and it compares well with the value 278.5 Kw obtained using paraxial method of Sodha *et al* [34]. As one moves away from the axial region for larger value of ρ, value of critical power increases (see Table 3). When $\rho = r_0 = 30$ μm (initial size of the Gaussian beam at plasma-vacuum interface), the critical power is 683.77 Kw, very large compared to paraxial method *i.e.* 2.46 times large. It is pointed out by various workers that the critical power calculated by aberration-less paraxial approximation is always three to four time less because the effects of nonlinear refraction are over-estimated here (5). Various correction had been suggested to overcome such problems associated with the paraxial methods. Central to the corrected paraxial approximation is to account for the plasma density or refractive index correction (in space) with the electromagnetic wave since both are transversely inhomogeneous. For laser beam in the transverse direction, the nonlinear dielectric inhomogeneity perceived by the wave cannot be approximated using simple Taylor expansion in the transverse cylinderical coordinate r. This is the reason why the paraxial theories fail in predicting the correct value of critical power and other important parameters related to self-focusing. Methods based on the invariants of nonlinear Schröndinger equation (NLSE) such as moment method originally developed by Vlasov *et al* [24] and later generalised by Lam *et al* [26] as well as variation method of Anderson and Bonnedab [28] has proved to be quite useful in estimating the self-focusing effect. These methods are supposed to be equivalent to a corrected paraxial theory because they intrinsically take care of the approximations. The fact that the value of critical power predicted by the present analysis is reasonably consistent with the values estimated by moments, variational theory and numerical calculations, makes the present analysis additionally useful.

Variation of dimensionless normalised self-trapped radius ($\omega_p r_0/c$) with βE_0^2 is given in Figure 6 using eq. (22).

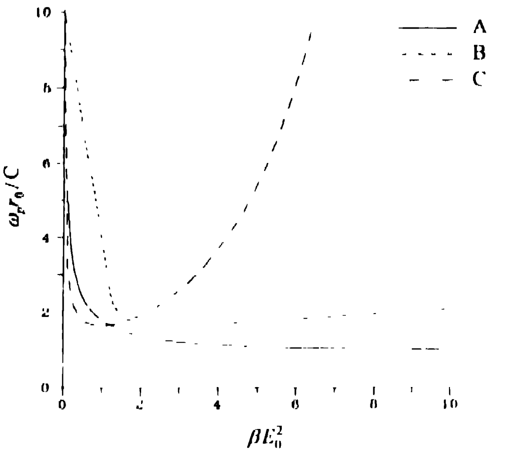

Figure 6. Dependence of normalised beam radius ($\omega_p r_0/C$) on incident beam intensity parameter (βE_0^2) Curve A for present analysis, Curve B for variational method and Curve C for paraxial ray method $r_0 = 30$ µm

The self-trapped radius for ponderomotive nonlinearity in plasma due to variational method is given as [27]

$$\left.\frac{\omega_p r_0}{c}\right|_{\text{variational}} = \left[\frac{\beta E_0^2}{E(\beta E_0^2) + \exp(-\beta E_0^2) - 1}\right]^{\frac{1}{2}}, \quad (23)$$

where $E(x)$ is a standard exponential integral and given as

$$E(x) = \int_0^x \frac{1 - \exp(-y)}{y} dy.$$

However from paraxial approximation method, self-trapped radius is [21]

$$\left.\frac{\omega_p r_0}{c}\right|_{\text{paraxial}} = \frac{\exp(\beta E_0^2/2)}{(\beta E_0^2/2)}. \quad (24)$$

Self-trapped beam radius employing eqs. (23) and (24) with beam intensity, has also been plotted for variational as well as for paraxial theory in Figure 6 (curves B and C respectively).

The saturation behaviour of the trapped radius or the nonlinearity in the high intensity region is an important phenomenon for self-trapping process. Results in Figure 6 clearly show that the present analysis gives much flatter curve in the saturation region as compared to paraxial method. Observed behaviour demonstrates almost same dependence as that follows from the variational method. Such type of variation is physically expected for self-trapped beam because under self-trapped condition, the plasma is completely expelled from the beam region and beam drills a 'clear hole' in plasma. Laser photons propagate freely down the empty pipe formed by this cavity in the plasma. Thus, the radius of photon pipe should be nearly independent of the intensity or power of beam, as predicted by the present analysis.

Finally, the results of our analysis and comparison with paraxial ray approximation as well as with the method of moments and variational theory demonstrates its advantages. It successfully provides a qualitative description of the beam characteristics in the nonlinear media in a more accurate way as compared to many popular paraxial ray methods.

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